OCR
Oxford Cambridge and RSA

Monday 26 June 2017 – Afternoon

A2 GCE MATHEMATICS (MEI)

4764/01 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4764/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (24 marks)

1 A car moves horizontally in a straight line with speed v at time t. The total resistance force on the car has magnitude kv where k is a positive constant. The car is powered by a rocket, which ejects burnt fuel backwards at a constant mass rate λ and at a constant speed u relative to the car. The initial mass of the car and the fuel is M and at time t, when some fuel still remains to be burnt, the mass of the car and the remaining fuel is m.

(i) Derive the differential equation $m\frac{\mathrm{d}v}{\mathrm{d}t}+u\frac{\mathrm{d}m}{\mathrm{d}t}=-kv$. **[3]**

(ii) Given that the initial speed of the car is zero, show that

$$v=\frac{\lambda u}{k}\left(1-\left(\frac{M-\lambda t}{M}\right)^{\frac{k}{\lambda}}\right),$$

and hence show that for small values of t the speed of the car is approximately $\frac{\lambda ut}{M}$. **[9]**

2 A particle of mass 3 kg moves along the x-axis by means of a driving force applied in the positive x-direction. There are no other forces acting on the particle. When the particle is x m from the origin O, its velocity is $v\,\mathrm{m\,s^{-1}}$. Initially $v = 3$ and the particle is at O. The magnitude of the driving force is F N, where $F=\mathrm{e}^{0.1x}\left(v^2-1\right)^{\frac{1}{3}}$.

(i) By solving a suitable equation of motion satisfied by the particle, show that F may be written as

$$F=\frac{2}{3}\mathrm{e}^{0.1x}\sqrt{10\mathrm{e}^{0.1x}-1}\,. \quad \textbf{[9]}$$

(ii) By using the work-energy principle, and without further integration, show that

$$\int_0^{10}\mathrm{e}^{0.1x}\sqrt{10\mathrm{e}^{0.1x}-1}\,\mathrm{d}x=k\left(\left(10\mathrm{e}-1\right)^{\frac{3}{2}}-27\right),$$

stating the exact value of the constant k. **[3]**

Section B (48 marks)

3 Fig. 3 shows a smooth wire in the form of a semi-circle with centre O and radius a. The wire is fixed in a vertical plane. The points C and D are at the ends of the wire at the same horizontal level as O. A small ring, P, of mass λm can move freely on the wire. One end of a light inextensible string of length $2a$ is attached to P. The string passes over a small smooth fixed pulley at C; a particle of mass μm hangs freely from its other end, vertically below C. One end of a second light inextensible string of length $2a$ is attached to P. This string passes over a small smooth fixed pulley at D; a particle of mass μm hangs freely from its other end, vertically below D. The radius OP makes an angle 2θ with the downward vertical, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$, and λ and μ are positive constants.

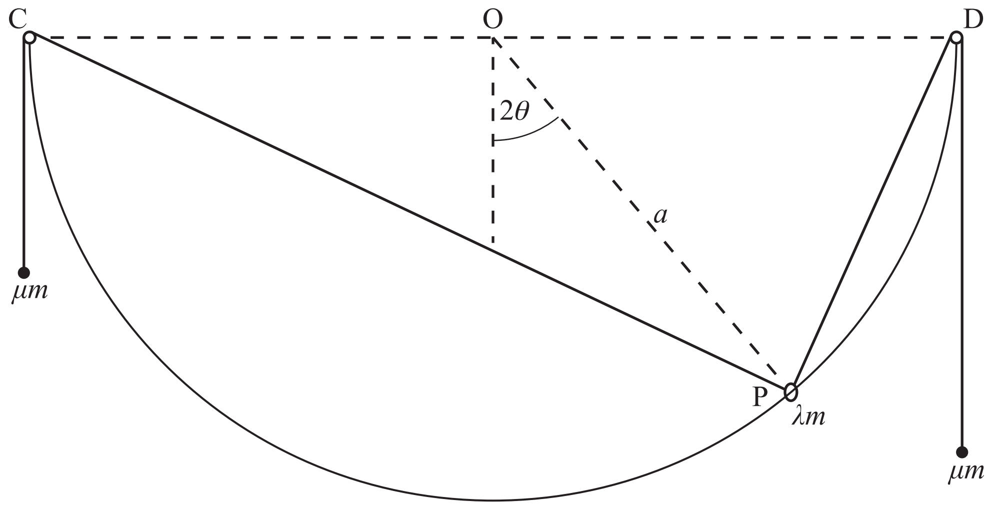

Fig. 3

(i) Find the potential energy, V, of the system relative to the level of CD, and hence show that

$$\frac{dV}{d\theta} = 2mga(\lambda \sin 2\theta - \sqrt{2}\mu \sin\theta).$$ **[8]**

(ii) Show that there are three values of θ for which the system is in equilibrium provided that $\lambda < \mu < \sqrt{2}\lambda$. **[5]**

(iii) Given that there are three positions of equilibrium, establish whether each of these positions is stable or unstable. **[8]**

You are now given that $\mu = 6$ and $\lambda = 3\sqrt{2}$.

(iv) Investigate the stability of the single equilibrium position of the system. **[3]**

4 A triangular lamina OAB of mass M kg has OA = OB and AB = $2a$ m. OX = $3a$ m, where X is the mid-point of AB. Fig. 4 shows this lamina in an x–y plane with origin O and OX horizontal. The mass per unit area ρ kg m^{-2} of the lamina is given by $\rho = k\left(1+\frac{x}{a}\right)$ where x m is the horizontal distance from O and k is a positive constant.

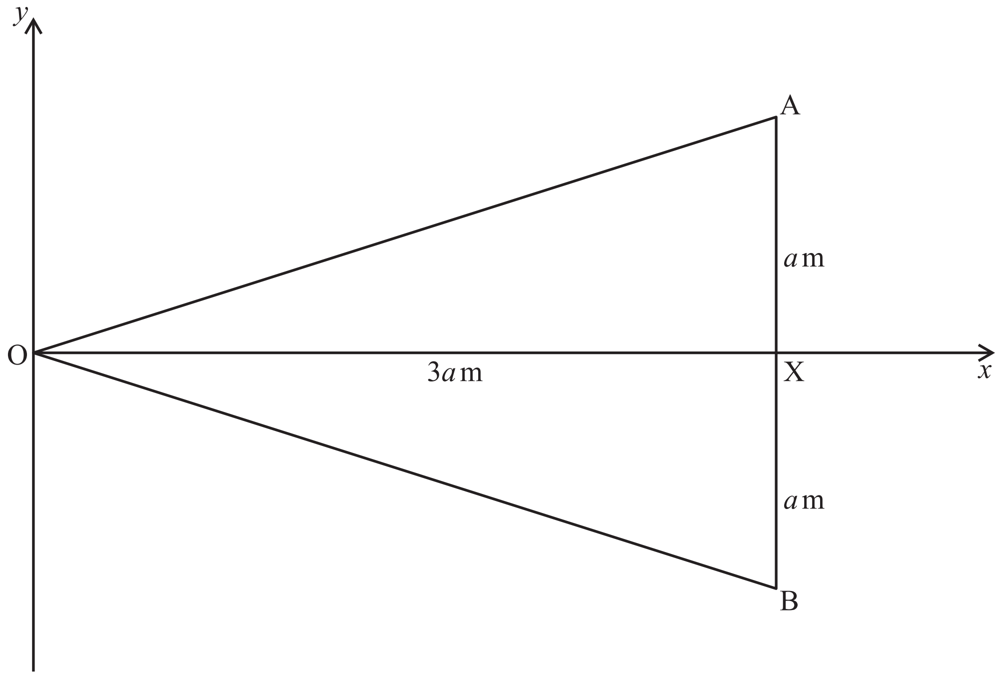

Fig. 4

(i) Show that $M = 9ka^2$. **[5]**

(ii) Show, using integration, that the moment of inertia of the lamina about an axis through O perpendicular to the plane of the lamina is $\frac{238}{45}Ma^2$. [You may assume the standard formula for the moment of inertia of a thin rod about an axis through its centre perpendicular to the rod.] **[7]**

The lamina is free to rotate in a vertical plane about a fixed smooth horizontal axis through O perpendicular to the lamina. The lamina is released from rest with OX making an angle ϕ with the downward vertical. At time t s after the lamina is released, OX makes an angle θ with the downward vertical.

(iii) Show that the angular velocity $\dot{\theta}$ of the lamina when it has turned through an angle θ satisfies

$$a\dot{\theta}^2 = \sigma g(\cos\theta - \cos\phi),$$

stating the exact value of the constant σ. **[8]**

You are now given that $a = 2.25$ and that ϕ is small.

(iv) Show that the motion is approximately simple harmonic, and find the approximate time when the lamina first comes instantaneously to rest. **[4]**

END OF QUESTION PAPER

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Candidate forename		Candidate surname	

Centre number						Candidate number				

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Section A (24 marks)

1 (i)	
1 (ii)	
	(answer space continued on next page)

1 (ii)	**(continued)**

2 (i)

(answer space continued on next page)

2 (i)	**(continued)**
2 (ii)	

Section B (48 marks)

3 (i)	

3 (ii)	
3 (iii)	**(answer space continued on next page)**

3 (iii)	**(continued)**
3 (iv)	

4 (i)

4 (ii)	

4 (iii)	

4 (iv)

Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4764**: Mechanics 4

Advanced GCE

Mark Scheme for June 2017

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✘	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Mechanics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

When a value is given in the paper
Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

When a value is not given in the paper
Accept any answer that agrees with the correct value to 2 s.f.

ft should be used so that only one mark is lost for each distinct error made in the accuracy to which working is done or an answer given. Refer cases to your Team Leader where the same type of error (e.g. errors due to premature approximation leading to error) has been made in different questions or parts of questions.

There are some mistakes that might be repeated throughout a paper. If a candidate makes such a mistake, (eg uses a calculator in wrong angle mode) then you will need to check the candidate's script for repetitions of the mistake and consult your Team Leader about what penalty should be given.

There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	Guidance
1	(i)		$(m+\delta m)(v+\delta v)-mv-\delta m(v-u)=-kv\delta t$	M1	Impulse = change in momentum – all three terms on lhs (condone F for $-kv$)
			$m\dfrac{\delta v}{\delta t}+u\dfrac{\delta m}{\delta t}+\dfrac{\delta m}{\delta t}\dfrac{\delta v}{\delta t}\delta t=-kv$	M1	Form differential equation (not dependent on previous M mark)
			$m\dfrac{\mathrm{d}v}{\mathrm{d}t}+u\dfrac{\mathrm{d}m}{\mathrm{d}t}=-kv$	E1	Complete argument (all working correct) – must give convincing reason if changing $-\delta m$ to δm
				[3]	
1	(ii)		$m=M-\lambda t$	B1	
			$(M-\lambda t)\dfrac{\mathrm{d}v}{\mathrm{d}t}+u(-\lambda)=-kv\Rightarrow\int\dfrac{\mathrm{d}v}{\lambda u-kv}=\int\dfrac{\mathrm{d}t}{M-\lambda t}$	M1	Substitute m , $\dfrac{\mathrm{d}m}{\mathrm{d}t}$ and separate variables
				M1	Integrate
			$-\dfrac{1}{k}\ln(\lambda u-kv)=-\dfrac{1}{\lambda}\ln(M-\lambda t)(+c)$	A1	
			$t=0, v=0\Rightarrow c=\dfrac{1}{\lambda}\ln M-\dfrac{1}{k}\ln(\lambda u)$	A1	Use conditions to obtain c
				M1	Attempt to make v the subject (using correct log laws)
			$v=\dfrac{\lambda u}{k}\left(1-\left(\dfrac{M-\lambda t}{M}\right)^{\frac{k}{\lambda}}\right)$	E1	
			$v=\dfrac{\lambda u}{k}\left(1-\left(1-\dfrac{k}{\lambda}\left(\dfrac{\lambda t}{M}\right)+\cdots\right)\right)$	M1	Binomial expansion of v
			$v\approx\dfrac{\lambda ut}{M}$	E1	
				[9]	

Question		Answer	Marks	Guidance
2	(i)	$3v\frac{dv}{dx} = e^{0.1x}\left(v^2 - 1\right)^{1/3}$	M1	For use of N2L with any expression for a
		$3\int \frac{v\,dv}{\left(v^2-1\right)^{1/3}} = \int e^{0.1x}dx$	M1	Separate variables
			M1	Attempt to integrate at least one term
		$\frac{9}{4}\left(v^2-1\right)^{2/3} = 10e^{0.1x} + c$	A1A1	A1 for one term correct, must have $+c$ for both A marks – if substitution used then must be correct in terms of original variables
		$x = 0, v = 3 \Rightarrow c = -1$	M1	Use conditions to obtain c (must show evidence of sub. for both x and v)
		$\left(v^2-1\right)^{1/3} = \frac{2}{3}\left(10e^{0.1x} - 1\right)^{1/2}$	M1	Attempt to make $\left(v^2-1\right)^{1/3}$ the subject
			M1	Substitute their $\left(v^2-1\right)^{1/3}$ into F
		$F = \frac{2}{3}e^{0.1x}\sqrt{10e^{0.1x} - 1}$	E1	Complete argument – must be sufficient working to indicate that result has been derived accurately
			[9]	
2	(ii)	$v^2 = \frac{8}{27}\left(10e - 1\right)^{3/2} + 1$	M1	Attempt to find v^2 at $x = 10$
		$\frac{2}{3}\int_0^{10} e^{0.1x}\left(10e^{0.1x} - 1\right)^{1/2} dx = \frac{1}{2}(3)\left(v^2 - 9\right)$	M1	Use $\int F dx = \frac{1}{2}m\left(v^2 - u^2\right)$
		$\int_0^{10} e^{0.1x}\sqrt{10e^{0.1x} - 1}\, dx = \frac{2}{3}\left(\left(10e - 1\right)^{\frac{3}{2}} - 27\right)$	A1	$k = \frac{2}{3}$
			[3]	

Question			Answer	Marks	Guidance
3	(i)		$-\lambda mga\cos 2\theta$	B1	GPE for ring
			$-\mu mg\left(2a-2a\sin\Theta\right)-\mu mg\left(2a-2a\sin\Phi\right)$	M1	GPE for both particles for any Θ,Φ (allow trig confusion). Note that $\text{DP}=2a\cos\left(\pi/4+\theta\right)=a\sqrt{2}\left(\cos\theta-\sin\theta\right)$ and $\text{CP}=2a\sin\left(\pi/4+\theta\right)=a\sqrt{2}\left(\cos\theta+\sin\theta\right)$
			$V=-\lambda mga\cos 2\theta-2\mu mga\left(1-\sin\left(\pi/4+\theta\right)\right)-2\mu mga\left(1-\sin\left(\pi/4-\theta\right)\right)$ or $-\lambda mga\cos 2\theta-\mu mga\left(2-\sqrt{2(1-\sin 2\theta)}\right)-\mu mga\left(2-\sqrt{2(1+\sin 2\theta)}\right)$	B1 A1	GPE for system (unsimplified) – B1 for ring and $-4\mu mga$
				M1	Compound angle formulae applied/trig. simplification and attempt to simplify (may appear after differentiation)
			$V=-\lambda mga\cos 2\theta-2\mu mga-2\mu mga+\cdots$ $\cdots+\sqrt{2}\mu mga\left(\cos\theta+\sin\theta\right)+\sqrt{2}\mu mga\left(\cos\theta-\sin\theta\right)$	A1	Correct expression in terms of θ
				M1	Differentiate
			$\dfrac{\mathrm{d}V}{\mathrm{d}\theta}=2mga\left(\lambda\sin 2\theta-\sqrt{2}\mu\sin\theta\right)$	E1	
				[8]	
3	(ii)		$\lambda\sin 2\theta-\sqrt{2}\mu\sin\theta=0$	M1	Set $V'=0$ and attempt to simplify
			$\sin\theta\left(2\lambda\cos\theta-\sqrt{2}\mu\right)=0$	M1	Use double angle formula and factorise
			$\sin\theta=0\Rightarrow\theta=0$ so there is always one equilibrium position (which is independent of λ and μ)	E1	Correctly showing equilibrium position at $\theta=0$
			$-\dfrac{\pi}{4}<\theta<\dfrac{\pi}{4}\Rightarrow\dfrac{\sqrt{2}}{2}<\cos\theta<1$ and $\cos\theta=\dfrac{\sqrt{2}\mu}{2\lambda}$	M1	Consider an expression for $\cos\theta$ in the correct interval
			So there are two further equilibrium positions provided that $\lambda<\mu<\sqrt{2}\lambda$ making three in total	E1	
				[5]	

Question			Answer	Marks	Guidance
3	(iii)			M1	Differentiate to find $V^{//}$
			$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}=4\lambda mga\cos(2\theta)-2\sqrt{2}\mu mga\cos\theta$	A1	
			$\theta=0\Rightarrow\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}=2mga\left(2\lambda-\sqrt{2}\mu\right)$	M1	Substitute $\theta=0$
			As $\lambda<\mu<\sqrt{2}\lambda$, $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}>0\Rightarrow$ stable	A1	Correct evaluation and conclusion
			$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}=4\lambda mga\left(2\cos^2\theta-1\right)-2\sqrt{2}\mu mga\cos\theta$	M1	Re-write $V^{//}$ using double angle formulae
			When $\theta=\pm\arccos\left(\frac{\mu\sqrt{2}}{2\lambda}\right)$ there are two equilibrium positions but $V^{//}$ will have the same sign for both of these values of θ $\because$ $V^{//}$ is an even function (of $\cos\theta$ only) therefore these two equilibrium positions will have the same stability	B1	Must have $\pm$ and conclude that both positions would have the same equilibrium
			$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}=2mga\left(\frac{\mu^2-2\lambda^2}{\lambda}\right)$ or $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}=-4mga\lambda\sin^2\theta$	M1	Substitute $\cos\theta=\frac{\mu\sqrt{2}}{2\lambda}$ and simplify to get an expression in terms of λ and μ or in terms of $\sin^2\theta$ (implies previous M mark)
			As $\lambda<\mu<\sqrt{2}\lambda$, $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}<0\Rightarrow$ unstable	A1	Correct evaluation and conclusion
				[8]	
3	(iv)		$\frac{\mathrm{d}V}{\mathrm{d}\theta}=12\sqrt{2}mga\sin\theta(\cos\theta-1)$	B1	oe
				M1	Use first derivative test
			When $\theta=0^-,\frac{\mathrm{d}V}{\mathrm{d}\theta}>0$ **and** when $\theta=0^+,\frac{\mathrm{d}V}{\mathrm{d}\theta}<0$ $\therefore$ unstable	A1	CAO
				[3]	

Question		Answer	Marks	Guidance
4	(i)	$\delta m = \rho(2y\delta x)$	B1	
		$M = k\int_0^{3a} 2\left(\frac{x}{3}\right)\left(1+\frac{x}{a}\right)dx$	B1	
		$M = \frac{2k}{3}\int x + \frac{x^2}{a}\,dx = \ldots$	M1	Simplify and attempt to integrate (limits not required)
		$M = \frac{2k}{3}\left[\frac{x^2}{2}+\frac{x^3}{3a}\right]_0^{3a} = \frac{2k}{3}\left(\frac{9a^2}{2}+9a^2\right)$	M1	Use of correct limits on their integrated expression
		$M = 9ka^2$	E1 **[5]**	
4	(ii)	About centre of strip $\delta I = \frac{1}{3}(2\rho y\delta x)y^2$	B1	
		By the parallel axis theorem, about the given axis	M1*	
		$\frac{2}{3}\rho y^3\delta x + 2\rho x^2 y\delta x$	A1	
		$I = \frac{2k}{3}\int_0^{3a}\left(1+\frac{x}{a}\right)\left(\frac{x^3}{27}+x^3\right)dx = \frac{56k}{81}\int_0^{3a} x^3+\frac{x^4}{a}\,dx$	M1dep*	Limits not required
		$= \frac{56k}{81}\left[\frac{x^4}{4}+\frac{x^5}{5a}\right]_0^{3a}$	M1	Integrating and stating correct limits – dependent on both previous M marks
		$= \frac{238}{5}ka^4$	A1	
		$= \frac{238a^2}{5}\left(\frac{M}{9}\right) = \frac{238}{45}Ma^2$	E1 **[7]**	At least one line if intermediate working before given answer

Question		Answer	Marks	Guidance
4	(iii)	$M\bar{x} = \int x\mathrm{d}m = \int_0^{3a} k\left(1+\frac{x}{a}\right)\left(\frac{2x}{3}\right)x\mathrm{d}x$	B1	Condone lack of limits
		$= \frac{2k}{3}\int_0^{3a} x^2 + \frac{x^3}{a}\mathrm{d}x = \frac{2k}{3}\left[\frac{x^3}{3}+\frac{x^4}{4a}\right]_0^{3a}$	M1	Integrating and using correct limits
		$= \frac{39}{2}ka^3$	A1	
		$\bar{x} = \frac{39ka^3}{2}\left(\frac{1}{9ka^2}\right) = \frac{13a}{6}$	A1	
			M1	Conservation of energy
		$\frac{1}{2}\left(\frac{238}{45}Ma^2\right)\dot{\theta}^2 = \ldots$	A1	
		$\cdots = Mg\left(\frac{13a}{6}\right)(\cos\theta - \cos\phi)$	A1ft	Follow cv($\bar{x}$) – dependent on first M mark
		$a\dot{\theta}^2 = \sigma g(\cos\theta - \cos\phi)$ where $\sigma = \frac{195}{238}$	A1	
			[8]	
4	(iv)	$2\frac{\mathrm{d}\theta}{\mathrm{d}t}\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = \frac{195(9.8)}{238(2.25)}\left(-\sin\theta\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)$	M1 A1	Differentiates their $\dot{\theta}^2$ wrt t Allow unsimplified and in terms of g
		ϕ small $\Rightarrow \sin\theta \approx \theta \therefore \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{91}{51}\theta \approx 0$ so approx.. SHM	A1ft	Allow in terms of g – must state SHM
		$t \approx \frac{\pi}{\sqrt{\frac{91}{51}}} = 2.35$ (3sf)	A1	2.3518751…
			[4]	

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Unit level raw mark and UMS grade boundaries June 2017 series

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AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)											
				Max Mark	a	b	c	d	e	u
4751	01	C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	58	53	49	45	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	55	49	44	39	34	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	54	49	45	41	36	0
4753	02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – MEI Applications of advanced mathematics (A2)	Raw	90	67	61	55	49	43	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	57	52	47	42	38	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	65	58	52	46	40	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	64	56	48	41	34	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	56	50	44	37	0
4758	02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – MEI Mechanics 1 (AS)	Raw	72	57	49	41	34	27	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – MEI Mechanics 2 (A2)	Raw	72	56	48	41	34	27	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – MEI Mechanics 3 (A2)	Raw	72	58	50	43	36	29	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – MEI Mechanics 4 (A2)	Raw	72	53	45	38	31	24	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – MEI Statistics 1 (AS)	Raw	72	61	55	49	43	37	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – MEI Statistics 2 (A2)	Raw	72	56	50	45	40	35	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – MEI Statistics 3 (A2)	Raw	72	63	57	51	46	41	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – MEI Decision mathematics 1 (AS)	Raw	72	52	46	41	36	31	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – MEI Decision mathematics 2 (A2)	Raw	72	53	48	43	39	35	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	58	53	48	43	37	0
4776	02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – MEI Numerical computation (A2)	Raw	72	55	48	41	34	27	0

		UMS	100	80	70	60	50	40	0
4798	01 FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
		UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241	01 Statistics 1 MEI (Z1)	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
G242	01 Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243	01 Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0

GCE Quantitative Methods (MEI)

			Max Mark	a	b	c	d	e	u
G244	01 Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02 Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
G245	01 Statistics 1 MEI	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
G246	01 Decision 1 MEI	Raw	72	52	46	41	36	31	0
		UMS	100	80	70	60	50	40	0

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Level 3 Certificate and FSMQ raw mark grade boundaries June 2017 series

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Level 3 Certificate Mathematics for Engineering

				Max Mark	a*	a	b	c	d	e	u
H860	01	Mathematics for Engineering		This unit has no entries in June 2017							
H860	02	Mathematics for Engineering									

Level 3 Certificate Mathematical Techniques and Applications for Engineers

				Max Mark	a*	a	b	c	d	e	u
H865	01	Component 1	Raw	60	48	42	36	30	24	18	0

Level 3 Certificate Mathematics - Quantitative Reasoning (MEI) (GQ Reform)

				Max Mark	a	b	c	d	e	u
H866	01	Introduction to quantitative reasoning	Raw	72	54	47	40	34	28	0
H866	02	Critical maths	Raw	60*	48	42	36	30	24	0
		*Component 02 is weighted to give marks out of 72	Overall	144	112	97	83	70	57	0

Level 3 Certificate Mathematics - Quantitive Problem Solving (MEI) (GQ Reform)

				Max Mark	a	b	c	d	e	u
H867	01	Introduction to quantitative reasoning	Raw	72	54	47	40	34	28	0
H867	02	Statistical problem solving	Raw	60*	41	36	31	27	23	0
		*Component 02 is weighted to give marks out of 72	Overall	144	103	90	77	66	56	0

Advanced Free Standing Mathematics Qualification (FSMQ)

				Max Mark	a	b	c	d	e	u
6993	01	Additional Mathematics	Raw	100	72	63	55	47	39	0

Intermediate Free Standing Mathematics Qualification (FSMQ)

				Max Mark	a	b	c	d	e	u
6989	01	Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0